1. \( a\mathbf{i} + b\mathbf{j} \) uniform electric field passes through a surface area \( \mathbf{A} \). Determine the electric flux through surfaces are;
   a) in the yz plane,
   b) in the xz plane,
   c) in the xy plane.

   The area vector is normal to the surface and point outward. So,

   a) \( \Phi_E = \mathbf{E} \cdot \mathbf{A} = (a\mathbf{i} + b\mathbf{j}) \cdot \mathbf{A} = aA \)
   b) \( \Phi_E = \mathbf{E} \cdot \mathbf{A} = (a\mathbf{i} + b\mathbf{j}) \cdot \mathbf{A} = bA \)
   c) \( \Phi_E = (a\mathbf{i} + b\mathbf{j}) \cdot \mathbf{A} = 0 \)
2. Consider an electric field in the constant direction is perpendicular to plane of a circle with radius $R$. The magnitude of electric field at a distance $r$ from the center of the circle is 

$$E_0 \left[1 - \frac{r}{R}\right]$$

Determine the electric flux through the circle.
3. Consider a closed triangular box resting within a horizontal electric field of magnitude $E=7.80\times10^4 \text{ (N/C)}$ as shown in Figure 1. Calculate the electric flux through 
   a) the vertical rectangular surface,  
   b) the slanted surface,  
   c) the entire surface of the box.

Figure 1

\[ \Phi_1 = EA_1 \cos \Theta_1 = 7.8 \times 10^4 (0.1 \times 0.3) \cos 180^\circ = -2.34 \text{ Nm}^2/\text{C} \]

\[ \Phi_2 = EA_2 \cos 60^\circ = 7.8 \times 10^4 (0.2 \times 0.3) \cos 60^\circ \]

\[ \Phi_2 = 2.34 \text{ Nm}^2/\text{C} \]

The flux through the base, the front, and the back surface of the box is zero. Because, the electric field vector is perpendicular to the surface.

\[ \Phi_{\text{net}} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 \]

\[ \Phi_{\text{net}} = -2.34 + 2.34 = 0 \text{ Nm}^2/\text{C} \]
4. A closed surface with dimensions \(a=0.2\,\text{m}\), \(b=0.3\,\text{m}\) and \(c=0.3\,\text{m}\) is located as in Figure 2. The left edge of the closed surface is located at position \(x=a\). The electric field throughout the region is nonuniform and given by \(E = (1+x^2)\,\text{N/C}\), where \(x\) is in meters.

a) Calculate the net electric flux leaving the closed surface.

b) What net charge is enclosed by the surface?

Figure 2

\[
\Phi_E = \Phi_1 + \Phi_2
\]

\[
\Phi_1 = \int_{x=a} (1+x^2)^{\frac{1}{2}} \, dA
\]

\[
\Phi_2 = \int_{x=a+c} \left[1+(a+c)^2\right]^{\frac{1}{2}} \, dA
\]

\[
\Phi_3 = \int_{x=a} \nabla \cdot \vec{E} \cdot d\vec{A}_1 + \int_{x=a+c} \nabla \cdot \vec{E} \cdot d\vec{A}_2
\]

\[
\Phi_4 = \int_{x=a} \left[1+(a+c)^2\right]^{\frac{1}{2}} \, d\vec{A}_3
\]

\[
\Phi_5 = \int_{x=a} \nabla \cdot \vec{E} \cdot d\vec{A}_4
\]

\[
\Phi_6 = \int_{x=a+c} \nabla \cdot \vec{E} \cdot d\vec{A}_5
\]

Same way,
\[ \Phi_E = - \int_{A_1} (1 + a^2) dA + \int_{A_2} \left[ 1 + (a + c)^2 \right] dA \]

\[ \Phi_E = -(1 + a^2) ab + \left[ 1 + (a + c)^2 \right] ab \]

\[ \Phi_E = -ab - a^3 b + ab + a^3 b + 2a^2 bc + abc = abc (2a + c) \]

\[
\begin{align*}
\begin{array}{c}
a = 0,2 \text{ m} \\
b = 0,3 \text{ m} \\
c = 0,3 \text{ m}
\end{array}
\end{align*}
\]

\[ \Phi_E = 12,6 \cdot 10^{-3} \text{ N m}^2/\text{C} \]

b) \[ \frac{\Phi_E}{\epsilon_0} \Rightarrow q = \epsilon_0 \frac{\Phi_E}{\epsilon_0} \]

\[ \epsilon_0 = 8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2 \]

\[ q = 8,85 \cdot 10^{-12} \cdot 12,6 \cdot 10^{-3} \]

\[ q_{\text{net}} = 1,12 \cdot 10^{-13} \text{ C} \]
5. Three infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure 3. The sheets have a uniform surface charge density $\sigma_1 = +5(\mu C/m^2)$, $\sigma_2 = -10(\mu C/m^2)$ and $\sigma_3 = +15(\mu C/m^2)$, respectively.

Calculate the electric field at
a) I zone,
b) II zone,
c) III zone,
d) IV zone.

![Figure 3](image-url)
II zone:
\[ \vec{E}_{II} = E_1(\hat{z}) + E_2(\hat{i}) + E_3(-\hat{i}) \]
\[ \vec{E}_{II} = (2.82 + 5.65 - 8.47) \times 10^5 \hat{z} \]
\[ \boxed{\vec{E}_{II} = 0} \]

III zone:
\[ \vec{E}_{III} = E_4(\hat{i}) + E_5(\hat{i}) + E_3(-\hat{i}) \]
\[ \vec{E}_{III} = (2.82 - 5.65 - 8.47) \times 10^5 \hat{i} \]
\[ \boxed{\vec{E}_{III} = 11.30 \times 10^5 \hat{i} \text{ (Nlc)}} \]

IV zone:
\[ \vec{E}_{IV} = E_4(\hat{i}) + E_5(\hat{i}) + E_3(\hat{i}) \]
\[ \vec{E}_{IV} = (2.82 - 5.65 + 8.47) \times 10^5 \hat{i} \]
\[ \boxed{\vec{E}_{IV} = 5.64 \times 10^5 \hat{i} \text{ (Nlc)}} \]
6. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire as in Figure 4. The wire has a charge per unit length of $+\lambda$, and the cylinder has a net charge per unit length of $+2\lambda$. From this information, use Gauss’s law to find the electric field in the regions:

a) $r<a$,

b) $a<r<b$,

c) $r>b$.

d) Determine the charge distribution of the cylindrical sheet.

**Figure 4**
c) \[ E(2\pi r L) = \frac{2\lambda + 2\lambda}{\varepsilon_0} \]
\[ E = \frac{1}{2\pi \varepsilon_0} \cdot \frac{3\lambda}{r} \]
\[ E = 6k \varepsilon \frac{2}{r} \quad \text{for } r > b \]

Because, wire induces the inner surface of the cylinder

\[ q_i = -2\lambda \]

\[ q\text{ cylinder} = q_i + q\text{ out} \]
\[ 2\lambda L = -2\lambda + q\text{ out} \]
\[ 2\lambda L + 2\lambda L = q\text{ out} \]
\[ q\text{ out} = 3\lambda L \]
7. A solid, insulating sphere of radius $R$ has a nonuniform charge density $\rho = \alpha r$ and a total charge $+2Q$ ($\alpha$ positive constant and $r$ radial distance from origin). Concentric with this sphere is a charged (+$4Q$), conducting shell sphere whose inner and outer radii are $2R$ and $3R$, as shown in Figure 5.

a) Find $\alpha$ constant in terms of $Q$ and $R$.

Find the magnitude of the electric field in the regions in terms of $k$, $Q$, $r$ and $R$.

b) $r < R$

c) $R < r < 2R$

d) $2R < r < 3R$

e) $r > 3R$

Figure 5
c) \[ E (4\pi r^2) = \frac{2Q}{\varepsilon_0} \]
\[ E = \frac{1}{4\pi\varepsilon_0} \frac{2Q}{r^2}, \quad E = 2k \frac{Q}{r^2} \quad R < r < 2R \]

E = 0 \quad 2R < r < 3R

\[ E (4\pi r^2) = \frac{2Q + 2Q}{\varepsilon_0} \]
\[ E = 6k \frac{Q}{r^2} \quad r > 3R \]

\[ q_{\text{tr}} = 2Q + (Q_{\text{tr}})_{\text{surface}} \quad 4Q-2Q \]