1. A uniform electric field of magnitude 325 V/m is directed in the negative y direction in Figure 1. The coordinates of point A are (-2,-3) m, and those of point B are (4, 5) m. Calculate the electric potential difference (V_B - V_A) using ACB and AB paths.

**Figure 1**

\[ V_B - V_A = \int_A \vec{E} \cdot d\vec{s} \]

**ACB path**

\[ V_B - V_A = - \int_A^{C} \vec{E} \cdot d\vec{s}_1 - \int_C^{B} \vec{E} \cdot d\vec{s}_2 \]

\[ V_B - V_A = - \int_A^{C} (-325 \hat{j}) \cdot d\vec{s}_1 - \int_C^{B} (-325 \hat{j}) \cdot d\vec{s}_2 \]

\[ V_B - V_A = 325 \int_A^{C} dy = 325 \left[ y \right]_{-3}^{5} = 325 [5 - (-3)] \]

\[ V_B - V_A = 2600 \text{ (V)} \]
AB path:

\[ V_B - V_A = - \int_A^B E \cdot d \vec{S} \]

\[ V_B - V_A = - \int_A^B (-125 \hat{j}) \cdot (dx \hat{i} + dy \hat{j}) \]

\[ V_B - V_A = 325 \int_A^B dy \]

\[ V_B - V_A = 325 \int_{-3}^{5} dy = 325 \left[ y \right]_{-3}^{5} \]

\[ V_B - V_A = 2600(V) \]
2. An electric dipole consists of two charges of $+5 \, \mu C$ and $-5 \, \mu C$ are placed at the points with coordinates $(-0.2; 0) \text{m}$ and $(0.2; 0) \text{m}$, as shown in Figure 2. A test charge of $q_0=3 \, \mu C$ is moved from the point $x=0.6 \text{m}$ to the point $x=-0.4 \text{m}$ with constant speed over a semicircle path intersecting y axis (radius of the path is $0.5 \text{m}$). How much work is done to move the test charge?

\[ W_{\text{adv}} = \Delta U = q_0 \Delta V = q_0 (V_a - V_b) \]

\[ W_{\text{adv}} = 3 \times 10^{-6} \left[ 150 000 - (-56250) \right] \]

\[ W_{\text{adv}} \approx 0.62 \, (J) \]
3. **a)** What is the net electric potential at the point A?

**b)** Calculate the energy required to assemble the array of charges shown in Figure 3, where a = 0.4 m, b = 0.4 m, and q = 6μC.

![Figure 3](image.png)

\[ V = \sum \frac{k q_i}{r_i} \]

\[ V_A = \frac{k q_1}{r_{12}} + \frac{k q_2}{r_{13}} + \frac{k q_3}{r_{23}} \]

\[ V_A = 3.10^3 \text{ (V)} \]

In an empty universe, a charge can be placed at its location with no energy investment.

To place \( q_1 \) charge there we must put in energy

\[ k \frac{q_1 q_2}{r_{12}} \]

Next, to bring up \( q_2 \) charge requires energy

\[ k \frac{q_1 q_3}{r_{13}} + k \frac{q_2 q_3}{r_{23}} \]

The total energy of the three charges is

\[ W = U = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \]

\[ W = 9.10^3 \left[ \frac{2.10^{-6}}{0.4} - 3 \frac{2.10^{-5}}{0.5} - 3 \frac{2.10^{-6}}{0.1} \right] \]

\[ W = -7.5 \text{ (J)} \]
4. A thin rod of length L and charge Q has a charge per unit length \( \lambda = \alpha y \) (\( \alpha \): cons.) lies along the y axis, as shown in Figure 4.

a) Calculate the electric potential at P point on x axis.

b) Obtain x component of the electric field at P point using the electric potential obtained in part (a).

c) If charge q is located at P point, calculate x component of the electric force exerted by the rod.

\[
\begin{align*}
\lambda &= \alpha y \\
V = k \int \frac{dq}{r} \\
V_{p} &= k \int \frac{dq}{r} = k \int \frac{\lambda \, dy}{\sqrt{x^2 + y^2}} \\
V_{p} &= k \int_{-L/4}^{L/4} \frac{\alpha y \, dy}{\sqrt{x^2 + y^2}} \\
V_{p} &= \frac{k \alpha}{2} \int \frac{dy}{\sqrt{x^2 + \frac{9L^2}{16}}} \\
V_{p} &= \frac{k \alpha}{2} \left[ \frac{x^2 + \frac{9L^2}{16}}{x^2 + \frac{L^2}{16}} \right] \\
V_{p} &= k \alpha \left( \sqrt{x^2 + \frac{9L^2}{16}} - \sqrt{x^2 + \frac{L^2}{16}} \right) \\

E &= -\nabla V = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) \\
E_{x} &= -\frac{\partial V}{\partial x} \quad \Rightarrow \quad E_{x} = -\frac{dV}{dx} \\
E_{x} &= -\frac{1}{\sqrt{x^2 + \frac{9L^2}{16}}} \left[ k \alpha \left( \sqrt{x^2 + \frac{9L^2}{16}} - \sqrt{x^2 + \frac{L^2}{16}} \right) \right]
\end{align*}
\]
\[ E_x = -k \alpha \left[ \frac{1}{2} \left( x^2 + \frac{9L^2}{16} \right) \frac{x}{L} - \frac{1}{2} \left( x^2 + \frac{L^2}{16} \right) \frac{x}{2L} \right] \]

\[ E_x = k \alpha \left( \frac{x}{\sqrt{x^2 + \frac{L^2}{16}}} - \frac{x}{\sqrt{x^2 + \frac{9L^2}{16}}} \right) \]

c) \[ F_E = q \vec{E} = q \left( E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \right) \]

\[ F_{p_x} = q \vec{E} \]

\[ F_{p_x} = k \alpha q \left( \frac{x}{\sqrt{x^2 + \frac{L^2}{16}}} - \frac{x}{\sqrt{x^2 + \frac{9L^2}{16}}} \right) \]
5. **a**) Calculate the electric potential at point \( P \) on the axis of the annulus shown in **Figure 5**, which has a uniform charge density \( \sigma \).

**b**) Find the electric field at point \( P \) using obtained the potential.

**Figure 5**

\[
V = k \int \frac{dq}{r}
\]

\[
V_e = k \int \frac{dq}{r} = \int \frac{\sigma dA}{\sqrt{x^2 + y^2}}
\]

\[
V_e = \pi k \sigma \int_0^{b} \frac{2\pi r dr}{\sqrt{r^2 + c^2}}
\]

\[
x^2 + r^2 = u
\]

\[
x^2 + b^2 = u
\]

\[
x = a; \quad u = x^2 + a^2
\]

\[
x = b; \quad u = x^2 + b^2
\]

\[
V_e = \pi k \sigma \int_0^{b} \frac{dr}{\sqrt{u}}
\]

\[
V_e = \pi k \sigma \left[ \frac{\sqrt{b^2}}{\sqrt{a^2}} \right] x^2
\]

\[
V_e = 2\pi k \sigma \left( \sqrt{x^2 + b^2} - \sqrt{x^2 + a^2} \right)
\]

**b**) \( E_x = \frac{dV}{dx} \)

\[
E_x = -\frac{d}{dx} \left[ 2\pi k \sigma \left( \sqrt{x^2 + b^2} - \sqrt{x^2 + a^2} \right) \right]
\]

\[
E_x = -2\pi k \sigma \left[ \frac{1}{2} \left( x^2 + b^2 \right)^{\frac{3}{2}} x - \frac{1}{2} \left( x^2 + a^2 \right)^{\frac{3}{2}} x \right]
\]

\[
E_x = 2\pi k \sigma \left( \frac{x}{\sqrt{x^2 + b^2}} - \frac{x}{\sqrt{x^2 + a^2}} \right)
\]
6. Consider two thin, conducting, spherical shells as shown in Figure 6. The inner shell has a radius \( r_1 = 15 \text{ cm} \) and a charge of 10 \( \text{nC} \). The outer shell has a radius \( r_2 = 30 \text{ cm} \) and a charge of \( -15 \text{ nC} \). Find 
   a) the electric field \( E \) and 
   b) the electric potential \( V \) in regions A, B, and C, with \( V = 0 \) at \( r = \infty \).

**Figure 6**

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_i}{\varepsilon_0} \]

**A region:**
\[ q_{iA} = 0; \quad E_A = 0 \quad (r < r_1) \]

**B region:**
\[ E_B = \frac{q_i}{4\pi\varepsilon_0 r^2} = \frac{k q_i}{r^2} \]
\[ E_B = 9 \times 10^9 \times \frac{10^{-9}}{r^2} \]
\[ E_B = \frac{90}{r^2} \quad (V/m) \quad (r_1 < r < r_2) \]

**C region:**
\[ E_C = \frac{q_i + q_c}{4\pi\varepsilon_0 r^2} = \frac{k (q_i + q_c)}{r^2} = 9 \times 10^9 \times \frac{(10 - 15) \times 10^{-9}}{r^2} \]
\[ E_C = -\frac{45}{r^2} \quad (V/m) \quad (r > r_2) \]
2nd Method:

\[ V_c - V_x = - \int_{x}^{c} \vec{E}_c \cdot d\vec{S}_c \]

\[ V_c = - \int_{x}^{c} E_c dS_c \cos \theta = - \int_{x}^{r_2} \frac{45}{r^2} (-dr) = + \int_{r_2}^{r_1} \frac{45}{r^2} dr = 45 \left| \frac{1}{r} \right|_{r_2}^{r_1} = - \frac{45}{r} (V) \]

\[ r = r_2 \]

\[ V_{r_2} = - \frac{45}{0.3} = -150(V) \]

\[ dS_2 = -dr \]

\[ V_a = - \int_{x}^{r_2} \vec{E}_c d\vec{S}_c - \frac{B}{\mu_0} \int_{r_2}^{r_1} \vec{E}_a d\vec{S}_a = - \int_{x}^{r_2} E_c dS_c \cos \theta - \frac{B}{\mu_0} \int_{r_2}^{r_1} E_a dS_a \cos \theta = - \int_{r_2}^{r_1} \frac{45}{r^2} (-dr) - \int_{r_2}^{r_1} \frac{90}{r^2} (-dr)(-1) = \]

\[ = + \int_{r_2}^{r_1} \frac{45}{r^2} dr - \int_{r_2}^{r_1} \frac{90}{r^2} dr = 45 \left| \frac{1}{r} \right|_{r_2}^{r_1} - 90 \left| \frac{1}{r} \right|_{r_2}^{r_1} = - \frac{45}{r_2} + 90 \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = - \frac{45}{0.3} + 90 \left( \frac{1}{0.3} - \frac{1}{r_2} \right) = -450 + \frac{90}{r} (V) \]

\[ r = r_1 \]

\[ V_{r_1} = -450 + \frac{90}{0.15} = 150(V) \]

\[ V_a = 150(V) \]
7. A solid insulating sphere of radius $R$ and charge $Q$ has a non-uniform charge density that varies with $r$ according to the expression $\rho = A r^2$, where $A$ is a constant and $r, R$ is measured from the center of the sphere. Use Gauss's law to

a) Determine the magnitudes of the electric fields outside and inside the sphere.

b) Determine the electric potential of a point inside the sphere.

c) Draw $E=f(r)$ and $V=f(r)$ graphs.

\[ E_{\text{out}} = \frac{A \epsilon_0 r^2}{4 \pi \epsilon_0 R^2} \]

\[ E_{\text{in}} = \frac{A \epsilon_0 r^2}{4 \pi \epsilon_0 R} \]

\[ V_A - V_\infty = V_A = -\int_{\infty}^{R} E_{\text{out}} \cdot ds - \int_{R}^{R} E_{\text{in}} \cdot ds = - \frac{A \epsilon_0}{4 \pi \epsilon_0} \left( \frac{4}{3} - 0 \right) = \frac{A}{\epsilon_0} \left( \frac{4}{3} - \frac{R^3}{4} \right) \]

\[ V_A = \frac{A \epsilon_0}{2 \epsilon_0} \left( 5 \epsilon_0^4 - \epsilon_0^4 \right) \]