



Name Surname

Registration No

Department

Group No Exam Hall

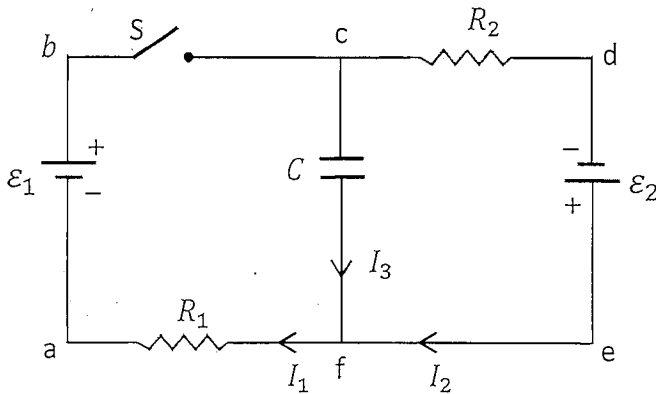
Signature of the Student

Lecturer's Name Surname

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PROBLEM 1

The switch  $S$  in the circuit is initially open and the capacitor is uncharged. In here,  $\epsilon_1 = 1V$ ,  $\epsilon_2 = 3V$ ,  $R_1 = 0.2\Omega$ ,  $R_2 = 0.3\Omega$  and  $C = 5\mu F$ .



a) Find the currents running in the circuit and the charge on the capacitor after a long time while  $S$  is still open.

$$I_1 = I_2 = I_3 = 0 \quad (3)$$

$$Q_i = C \Delta V_c \quad (1)$$

$$Q_i = C \cdot \epsilon_2 = (5\mu F)(3V)$$

$$Q_i = 15\mu C \quad (2)$$

b) Now the switch is closed. Find the currents and the charge on the capacitor after a long time after  $S$  is closed.

$$(1) \quad I_3 = 0 \quad ; \quad I_1 = I_2 = \frac{\epsilon_1 + \epsilon_2}{R_1 + R_2} \quad (2)$$

$$I_1 = I_2 = \frac{4}{0.5} = \frac{40}{5}$$

$$I_1 = I_2 = 8A \quad (2)$$

Charge; Loop abcfa:

$$\epsilon_1 + \Delta V_c - I_1 R_1 = 0 \quad (3)$$

$$\Delta V_c = I_1 R_1 - \epsilon_1 = 8(0.2) - 1$$

$$\Delta V_c = 0.6V$$

$$Q_f = C \cdot \Delta V_c = (5\mu F)(0.6)$$

$$Q_f = 3\mu C \quad (3)$$

c) Find powers supplied by the batteries and consumed across the resistors after switch  $S$  is closed for a long time.

$$P_{\epsilon_1} = \epsilon_1 I_1 = 1 \cdot 8 = 8W \quad (2)$$

$$P_{\epsilon_2} = \epsilon_2 I_2 = 3 \cdot 8 = 24W \quad (2)$$

$$P_E = P_{\epsilon_1} + P_{\epsilon_2} \Rightarrow P_E = 32W$$

Supplied power

$$P_{R_1} = I_1^2 R_1 = 8^2(0.2) = 12.8W \quad (2)$$

$$P_{R_2} = I_2^2 R_2 = 8^2(0.3) = 19.2W \quad (2)$$

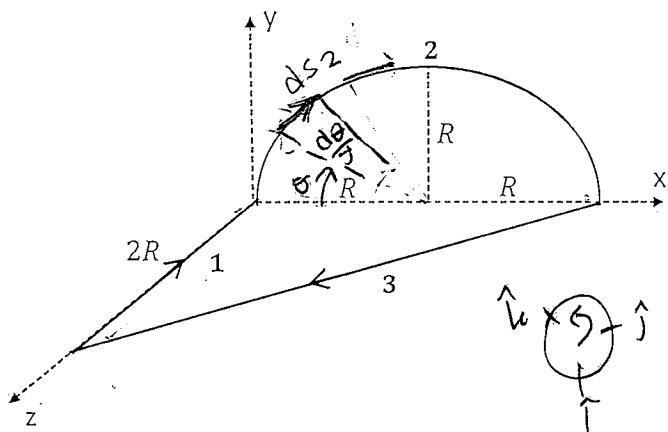
$$P_R = P_{R_1} + P_{R_2} = 12.8 + 19.2$$

$$P_R = 32W$$

Consumed power

PROBLEM 2

A closed loop carrying a constant current  $I$  is in a uniform magnetic field given by  $\vec{B} = B_0(\hat{i} + \hat{j})$  (T) as shown in figure ( $\pi = 3$ ).



a) Find the magnetic forces acting on each wire.

Wire 1:  $\vec{F}_1 = I \vec{l}_1 \times \vec{B}$ ;  $\vec{l}_1 = 2R(-\hat{k})$  (1)

$$\vec{F}_1 = 2IRB_0(-\hat{k}) \times (\hat{i} + \hat{j})$$

$$\vec{F}_1 = 2IRB_0(\hat{i} - \hat{j})$$
 (3)

Wire 2:  $\vec{F}_2 = I \vec{l}_2 \times \vec{B}$ ;  $\vec{l}_2 = 2R\hat{i}$  (2)

$$\vec{F}_2 = 2IRB_0\hat{i} \times (\hat{i} + \hat{j})$$

$$\vec{F}_2 = 2IRB_0\hat{k}$$
 (3)

Wire 3:  $\vec{F}_3 = I \vec{l}_3 \times \vec{B}$ ,  $\vec{l}_3 = 2R(-\hat{i} + \hat{k})$  (2)

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{k}) \times (\hat{i} + \hat{j})$$

$$\vec{F}_3 = 2IRB_0(-\hat{k} + \hat{j} - \hat{i})$$
 (3)

$$\vec{F}_3 = 2IRB_0(-\hat{i} + \hat{j} + \hat{k})$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 //$$

Alternative method for wire 2;

$$\vec{F}_2 = I \int d\vec{s}_2 \times \vec{B}; d\vec{s}_2 = R d\theta (\sin\theta \hat{i} + \cos\theta \hat{j})$$

$$\vec{F}_2 = IRB_0 \int_0^\pi (\sin\theta \hat{i} + \cos\theta \hat{j}) \times (\hat{i} + \hat{j}) d\theta$$

$$\vec{F}_2 = IRB_0 \int_0^\pi (\sin\theta - \cos\theta) d\theta \hat{k}$$

$$\vec{F}_2 = IRB_0 (-\cos\theta + \sin\theta) \Big|_0^\pi \hat{k}$$

$$\vec{F}_2 = 2IRB_0(\hat{k})$$

b) Find the magnetic dipole moment  $\vec{\mu}$  of the loop.

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2$$
 (1)

$$\vec{\mu}_1 = I \vec{A}_1 \Rightarrow \vec{\mu}_1 = I \frac{\pi R^2}{2} (-\hat{k})$$
 (2)

$$\vec{\mu}_2 = I \vec{A}_2 \Rightarrow \vec{\mu}_2 = I \frac{4R^2}{2} (-\hat{j})$$
 (2)

$$\vec{\mu} = \frac{3IR^2}{2}(-\hat{k}) + 2IR^2(-\hat{j})$$

$$\vec{\mu} = IR^2(-2\hat{j} - \frac{3}{2}\hat{k})$$
 (2)

c) Find the magnetic potential energy  $U$  of the loop.

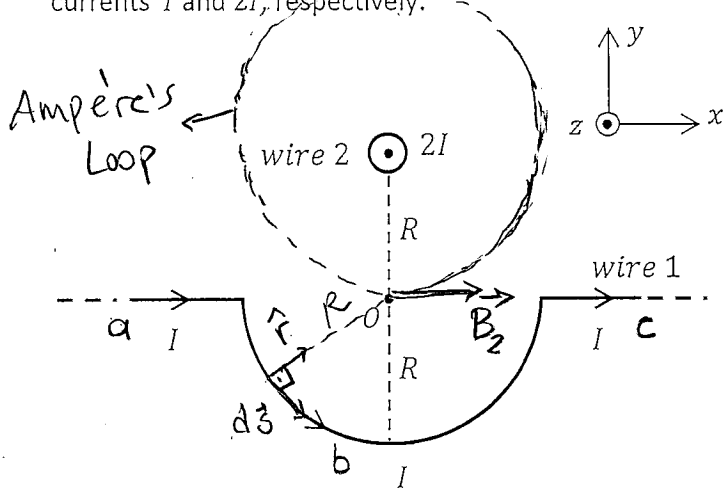
$$U = -\vec{\mu} \cdot \vec{B}$$
 (2)

$$U = -IR^2(2\hat{j} + \frac{3}{2}\hat{k}) \cdot B_0(\hat{i} + \hat{j})$$

$$U = 2IB_0R^2$$
 (2)

PROBLEM 3

An infinitely long current wire (wire 1) is bent into a semicircle of radius  $R$  and two semi-infinitely long straight wires. Wire 1 is placed on  $xy$ -plane as shown in figure. Another infinitely long straight wire (wire 2) is placed parallel to  $z$ -axis at  $y = R$ . Wire 1 and wire 2 are carrying currents  $I$  and  $2I$ , respectively.



a) Find the total magnetic field vector at point  $O$  due to the wire 1 and wire 2.

Wire 1: Since  $|d\vec{s} \times \vec{r}| = 0$  there is no contribution from segments  $a$  and  $c$ ;  $B_a = B_c = 0$  (2)

Segment b: Biot-Savart Law (1) or  $B = \frac{\mu_0 I}{4\pi} \int \frac{ds \sin\theta}{r^2}$   
 $B_b = \frac{\mu_0 I}{4\pi} \int \frac{|d\vec{s} \times \vec{r}|}{r^2}$ ;  $r = R$  (1)

$|d\vec{s} \times \vec{r}| = ds$  direction  $\rightarrow \odot \hat{k}$

$$B_b = \frac{\mu_0 I}{4\pi} \int \frac{ds}{R^2} = \frac{\mu_0 I}{4\pi R^2} \int ds$$

$$\vec{B}_b = \frac{\mu_0 I}{4R} \hat{k} \quad (1)$$

$$B = \frac{\mu_0 I}{4R} \quad (2)$$

$$\vec{B}_1 = \vec{B}_a + \vec{B}_b + \vec{B}_c$$

$$\vec{B}_1 = \frac{\mu_0 I}{4R} \hat{k} \quad (2)$$

Wire 2; Ampère's Law

$$\oint \vec{B}_2 \cdot d\vec{s} = \mu_0 I_{in} \quad (2)$$

$$B_2 \cdot 2\pi R = \mu_0 \cdot 2I \quad (2)$$

$$\vec{B}_2 = \frac{\mu_0 I}{\pi R} \hat{i} \quad (2)$$

$$\vec{B}_0 = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_0 = \frac{\mu_0 I}{R} \left( \frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right) \quad (3)$$

b) A charge  $+q$  is passing through point  $O$  with a velocity of  $\vec{v} = v_0 \hat{j}$ . Find the magnetic force acting on the charge.

$$\vec{F}_B = q \vec{v} \times \vec{B}_0 \quad (2)$$

$$\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \hat{j} \times \left( \frac{1}{\pi} \hat{i} + \frac{1}{4} \hat{k} \right)$$

$$\vec{F}_B = \frac{qv_0 \mu_0 I}{R} \left( \frac{1}{4} \hat{i} - \frac{1}{\pi} \hat{k} \right) \quad (3)$$

PROBLEM 4

A circular conductive loop of radius  $r$  is placed in a uniform magnetic field  $\vec{B} = B_0\hat{j}$  (figure 1). Loop rotates about z-axis with a constant angular speed  $\omega$  (figure 2). ( $\theta = \omega t$ )

Figure 1

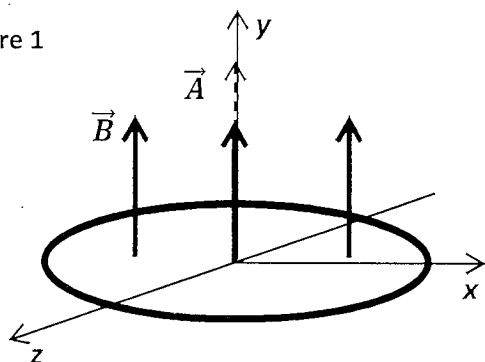
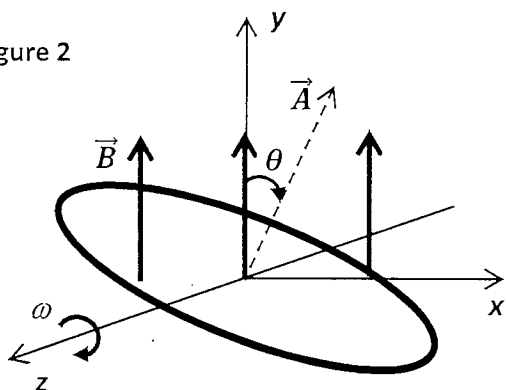


Figure 2



a) Find the electromotive force induced in the loop during the rotation.

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (2)$$

$$\Phi_B = B_0 A \cos\theta \quad (4), \quad \theta = \omega t$$

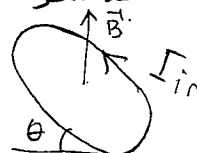
$$\Phi_B = B_0 A \cos\omega t$$

$$\mathcal{E} = - \frac{d}{dt} (B_0 A \cos\omega t) \quad (2)$$

$$\mathcal{E} = + B_0 A \omega \sin\omega t \quad (4)$$

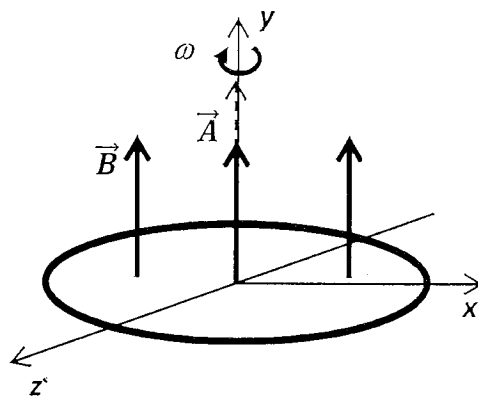
b) If the resistance of the loop is  $R$ , find the magnitude of the induced current. Explain the direction of the induced current when  $0 < \theta < 90^\circ$  according to the Lenz Law.

Since magnetic flux decreases when  $0 < \theta < 90^\circ$ ; induced current  $I_i$  must be counter clockwise in order to create magnetic field in the same direction (2)



$$I_i = \frac{|\mathcal{E}|}{R} \Rightarrow I_i = \frac{B_0 A \omega \sin\omega t}{R} \quad (3)$$

c) Find the electromotive force induced in the loop if it rotates about y-axis with a constant angular speed  $\omega$ .



In this case;  $\Phi_B = B_0 A \cos 0^\circ$  (2) and constant all the time. Therefore;

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = 0 \quad (5)$$