

RECITATION-1

1- Which of the equations below are dimensionally correct? (where v is velocity, x is position, a is acceleration, and t is time)

a) $x_s = x_i + v_{xi}t + \frac{1}{2}at^2$

b) $v_{xs}^2 = v_{xi}^2 - 2a(x_s - x_i)$

① a) $x_s = x_i + v_{xi}t + \frac{1}{2}at^2$
 $[L] = [L] + \frac{[L]}{[T]} \cancel{[T]} + \frac{[L]}{[T]^2} \cancel{[T]^2}$
 $[L] = [L] \quad \text{TRUE}$

b) $v_{xs}^2 = v_{xi}^2 - 2a(x_s - x_i)$
 $\frac{[L]^2}{[T]^2} = \frac{[L]^2}{[T]^2} - \frac{[L]}{[T]^2} [L]$
 $\frac{[L]^2}{[T]^2} = \frac{[L]^2}{[T]^2} \quad \text{TRUE}$

2-

a) By using $E = mc^2$ and $E = \frac{hc}{\lambda}$ expressions, find the dimension of the Planck constant and SI units. (in here, E is the energy, c is the speed of light, λ is wavelength, m is mass and h is the Planck constant)

$$mc^2 = \frac{hc}{\lambda} \rightarrow h = mc\lambda$$

$$m \text{ (mass)} : [M]; \quad c \text{ (speed of light)}: \frac{\Delta}{t} : [L]/[T]$$

$$\lambda \text{ (wavelength)} : [L]$$

$$[h] = [M][L][T]^{-1}[L]$$

$$[h] = [M][L]^2[T]^{-1} \rightarrow \text{SI} \rightarrow \text{kg m}^2 \text{ s}^{-1}$$

- b) The period of a simple pendulum of length "l" is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where g is the acceleration due to the gravity. Show that the equation is dimensionally correct. Find its unit in SI unit system.

$$[T] = \sqrt{\frac{[L]}{\frac{[L]}{[T]^2}}} = \sqrt{[T]^2} = [T]$$

In SI unit system, its unit is second (s)

- 3- In a rigid body, the distance between two adjacent atoms or molecules is assumed to be approximately equal to 2 times the radius of the volume of a molecule or atom. Find the distance between two adjacent atoms for;
- Iron and
 - Sodium.

(The densities of iron and sodium are given as $7,87 \text{ g/cm}^3$ and $1,013 \text{ g/cm}^3$, respectively. The atomic masses are also $9,27 \times 10^{-26} \text{ kg}$ and $3,82 \times 10^{-26} \text{ kg}$, respectively.)

- a) **Volume of an iron (Fe) atom**

$$V_{\text{Fe}} = \frac{4}{3} \pi r_{\text{Fe}}^3 = \frac{m_{\text{Fe}}}{\rho_{\text{Fe}}} \quad \text{ve yarıçapı } r_{\text{Fe}} = \left(\frac{3 m_{\text{Fe}}}{4 \pi \rho_{\text{Fe}}} \right)^{1/3}$$

$$r_{\text{Fe}} = \left(\frac{3 \times 9,27 \times 10^{-26} \text{ kg}}{4 \pi \times 7,87 \frac{10^{-3} \text{ kg}}{10^{-6} \text{ m}^3}} \right)^{1/3} = 1,41 \times 10^{-10} \text{ m}$$

Distance between two iron atom:

$$d_{\text{Fe}} = 2 \times r_{\text{Fe}} = 2,82 \times 10^{-10} \text{ m} \quad \text{bulunur.}$$

- b) **Distance between two sodium (Na) atom:**

$$d_{\text{Na}} = 2 \times r_{\text{Na}} = 2 \left(\frac{3 m_{\text{Na}}}{4 \pi \rho_{\text{Na}}} \right)^{1/3}$$

$$= 2 \left(\frac{3 \times 3,82 \times 10^{-26} \text{ kg}}{4 \pi \times 1,013 \times \frac{10^{-3} \text{ kg}}{10^{-6} \text{ m}^3}} \right)^{1/3} = 4,16 \times 10^{-10} \text{ m.}$$

- 4- Find the equivalent values in SI units :
 $1,0 \text{ g/cm}^3$, $980,0 \text{ cm/s}^2$, $9,1 \times 10^{-37} \text{ g}$, $1 \mu\text{m}$, $1,0 \text{ ms}$ and $1,0 \text{ ft}$.

$$1 \text{ g/cm}^3 = 1 \times \frac{10^{-3}}{10^{-6}} \frac{\text{kg}}{\text{m}^3} = 10^3 \text{ kg/m}^3$$

$$980 \text{ cm/s}^2 = 9,8 \text{ m/s}^2$$

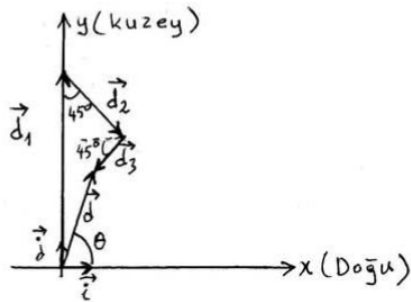
$$9,1 \times 10^{-27} \text{ g} = 9,1 \times 10^{-30} \text{ kg}$$

$$1 \mu\text{m} = 10^{-6} \text{ m}$$

$$1 \text{ ms} = 10^{-3} \text{ s}$$

$$1 \text{ ft} = 0,3048 \text{ m}$$

- 5- A novice golfer on the green takes three strokes to sink the ball. The successive displacements are 4.00 m to the north, 2.00 m south-east, and 1.00 m west of south. Starting at the same initial point, an expert golfer could make the hole in what single displacement?



$$\vec{d}_1 = 4\vec{j} \text{ (m)}$$

$$\vec{d}_2 = 2 \cos 45^\circ \vec{i} - 2 \sin 45^\circ \vec{j}$$

$$\vec{d}_3 = -1 \cdot \cos 45^\circ \vec{i} - 1 \cdot \sin 45^\circ \vec{j}$$

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \frac{\sqrt{2}}{2} \vec{i} + \left(4 - \frac{3}{2}\sqrt{2}\right) \vec{j}$$

$$|\vec{d}| = \sqrt{\frac{2}{4} + 16 - 12\sqrt{2} + \frac{9}{2}} \cong 2 \text{ m}$$

$$\tan \theta = \frac{4 - \frac{3}{2}\sqrt{2}}{\frac{\sqrt{2}}{2}} = 2,69 \quad \theta = 69,6^\circ$$

- 6- Two vectors are given as: $\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 4\hat{k}$. Find
- $\vec{a} + \vec{b}$ vector and its magnitude
 - $\vec{a} - \vec{b}$ vector and its magnitude
 - Find a vector \vec{c} that $\vec{a} - \vec{b} + \vec{c} = 0$

$$a) \quad \vec{c} = \vec{a} + \vec{b} = (4\hat{i} - 3\hat{j} + \hat{k}) + (-\hat{i} + 1\hat{j} + 4\hat{k}) = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

$$|\vec{c}| = \sqrt{3^2 + (-2)^2 + 5^2} = \sqrt{38}$$

$$b) \quad \vec{c} = \vec{a} - \vec{b} = (4\hat{i} - 3\hat{j} + \hat{k}) - (-\hat{i} + 1\hat{j} + 4\hat{k}) = 5\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{c}| = \sqrt{5^2 + (-4)^2 + (-3)^2} = \sqrt{50}$$

$$c) \quad \vec{a} + \vec{b} + \vec{c} = 0$$

$$(4\hat{i} - 3\hat{j} + \hat{k}) - (-\hat{i} + 1\hat{j} + 4\hat{k}) + (c_x\hat{i} + c_y\hat{j} + c_z\hat{k}) = (0\hat{i} + 0\hat{j} + 0\hat{k})$$

$$c_x = -5, \quad c_y = 4, \quad c_z = 3$$

7- **A**, **B** and **C** are defined as vectors and their components are given as: $A_x=3$, $A_y=-2$ and $A_z=2$, $B_x=0$, $B_y=0$, $B_z=4$, $C_x=2$, $C_y=-3$ and $C_z=0$. Find

$$a) \quad \vec{A} \cdot (\vec{B} + \vec{C})$$

$$\begin{aligned} \vec{B} + \vec{C} &= 2\hat{i} - 3\hat{j} + 4\hat{k} \\ \vec{A} \cdot (\vec{B} + \vec{C}) &= (3\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) \\ \vec{A} \cdot (\vec{B} + \vec{C}) &= 6 + 6 + 8 = \boxed{20} \end{aligned}$$

$$b) \quad \vec{A} \times (\vec{B} + \vec{C})$$

$$\begin{aligned} \vec{A} \times (\vec{B} + \vec{C}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 2 \\ 2 & -3 & 4 \end{vmatrix} = \hat{i}(-8+6) - \hat{j}(12-4) + \hat{k}(-9+4) \\ &= \boxed{-2\hat{i} - 8\hat{j} - 5\hat{k}} \end{aligned}$$

$$c) \quad \vec{A} \cdot (\vec{B} \times \vec{C})$$

$$\begin{aligned} \vec{B} \times \vec{C} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 4 \\ 2 & -3 & 0 \end{vmatrix} = \hat{i}(0+12) - \hat{j}(0-8) + \hat{k}(0-0) \\ &= 12\hat{i} + 8\hat{j} \\ \vec{A} \cdot (\vec{B} \times \vec{C}) &= (3\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (12\hat{i} + 8\hat{j}) = 36 - 16 = \boxed{20} \end{aligned}$$

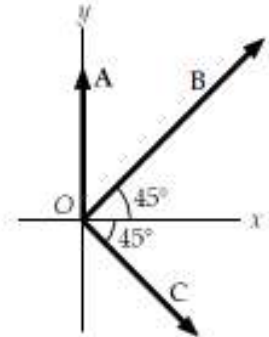
$$d) \quad \vec{A} \times (\vec{B} \times \vec{C})$$

$$\begin{aligned} \vec{A} \times (\vec{B} \times \vec{C}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 2 \\ 12 & 8 & 0 \end{vmatrix} = \hat{i}(0-16) - \hat{j}(0-24) + \hat{k}(24+24) \\ &= \boxed{-16\hat{i} + 24\hat{j} + 48\hat{k}} \end{aligned}$$

- 8- Three displacement vectors of a croquet ball are shown in Figure, where $|A| = 20.0$ units, $|B| = 40.0$ units, and $|C| = 30.0$ units. Find (a) the resultant in unitvector notation and (b) the magnitude and direction of the resultant displacement.

(a) $R_x = 40.0 \cos 45.0^\circ + 30.0 \cos 45.0^\circ = 49.5$
 $R_y = 40.0 \sin 45.0^\circ - 30.0 \sin 45.0^\circ + 20.0 = 27.1$
 $R = \boxed{49.5\hat{i} + 27.1\hat{j}}$

(b) $|R| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$
 $\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7^\circ}$



- 9- A person going for a walk and follows the path: First walks through northwest 4 km which makes 20 degree with the north. Then, 5 km in the way of north and lastly 3 km through the east. At the end of the walk, what is the person's resultant displacement measured from the starting point?

$\vec{d}_1 = 4(\cos 140^\circ \hat{i} + \sin 140^\circ \hat{j})$
 $\vec{d}_1 = 4(-0.342\hat{i} + 0.939\hat{j})$
 $\vec{d}_1 = -1.368\hat{i} + 3.758\hat{j} \text{ (km)}$

$\vec{d}_2 = 5\hat{j} \text{ (km)}$
 $\vec{d}_3 = 3\hat{i} \text{ (km)}$

$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$
 $\vec{d} = (-1.368 + 3)\hat{i} + (3.758 + 5)\hat{j}$
 $\vec{d} = 1.632\hat{i} + 8.758\hat{j} \text{ (km)}$

$|\vec{d}| = \sqrt{(1.632)^2 + (8.758)^2}$
 $|\vec{d}| = 8.907 \text{ km}$

$\tan \alpha = \frac{8.758}{1.632} = 5.366$
 $\alpha = \tan^{-1}(5.366)$
 $\alpha = 79.44^\circ$

10- A proton with velocity $\vec{v} = 1,0 \cdot 10^6 \hat{i} + 2,0 \cdot 10^6 \hat{j} - 2,0 \cdot 10^6 \hat{k}$ in a magnetic field which is given by $\vec{B} = 0,2 \hat{i} - 0,3 \hat{j} + 0,4 \hat{k}$. Find the force on proton using $\vec{F} = q\vec{v} \times \vec{B}$ expression. ($q = 1,6 \times 10^{-19} \text{ C}$).

$$a) \vec{v} = 1,0 \cdot 10^6 \hat{i} \text{ (m/s)} + 2,0 \cdot 10^6 \hat{j} \text{ (m/s)} - 2,0 \cdot 10^6 \hat{k} \text{ (m/s)}$$

$$\vec{B} = 0,2 \hat{i} \text{ (T)} - 0,3 \hat{j} \text{ (T)} + 0,4 \hat{k} \text{ (T)}$$

$$\boxed{\vec{F} = q \vec{v} \times \vec{B}}$$

$$\vec{v} \times \vec{B} = (0,2 \hat{i} - 0,8 \hat{j} - 0,7 \hat{k}) \cdot 10^6$$

$$\vec{F} = 1,6 \cdot 10^{-19} (0,2 \hat{i} - 0,8 \hat{j} - 0,7 \hat{k}) \cdot 10^6$$

$$\vec{F} = 0,32 \cdot 10^{-13} \hat{i} - 1,28 \cdot 10^{-13} \hat{j} - 1,12 \cdot 10^{-13} \hat{k} \text{ (N)}$$

$$\boxed{\vec{F} = (32 \hat{i} - 128 \hat{j} - 112 \hat{k}) \cdot 10^{-15} \text{ (N)}}$$