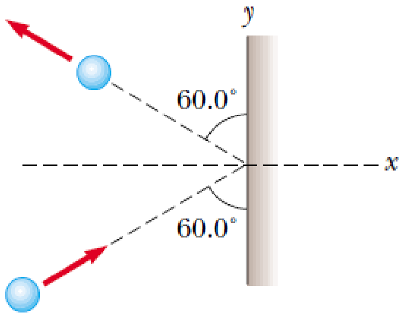
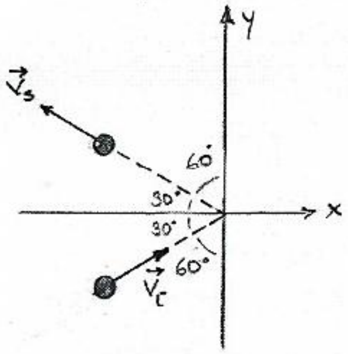
Recitation-5

Linear Momentum and Collisions

1- A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle as in Figure. If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball ?





$m = 3\ kg$

$\Delta t = 0{,}2\ s$

$V_i = 10\ m/s$

$V_s = 10\ m/s$

$\vec{V_i} = V_x \vec{i} + V_y \vec{j}$

$= V_i(\cos 30\,\vec{i} + \sin 30\,\vec{j})$

$\vec{V_s} = V_i(-\cos 30\,\vec{i} + \sin 30\,\vec{j})$

$\vec{P_i} = m \cdot \vec{V_i}$, $\vec{P_s} = m \cdot \vec{V_s}$

$\Delta \vec{P} = \vec{F} \cdot \Delta t$,

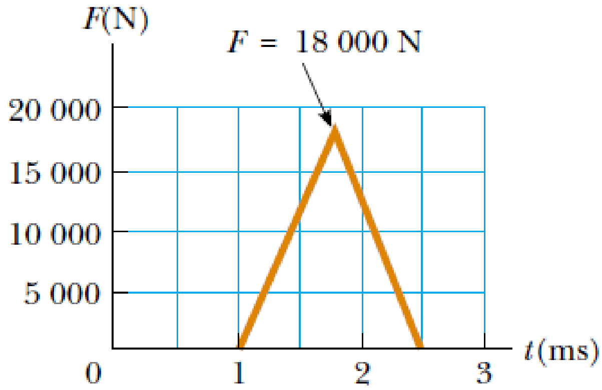
In the case of a constant Force:

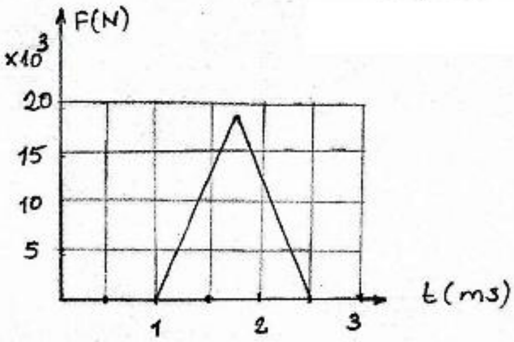
$$\Delta \vec{P} = \bar{\vec{F}} \cdot \Delta t \longrightarrow \bar{\vec{F}} = \frac{\Delta \vec{P}}{\Delta t} = \frac{\vec{P_s} - \vec{P_i}}{\Delta t} = \frac{m V_i \left[(-\cos 30\,\vec{i} + \sin 30\,\vec{j}) - (\cos 30\,\vec{i} + \sin 30\,\vec{j})\right]}{\Delta t}$$

$$\bar{\vec{F}} = \frac{-2 m V_i \cos 30\,\vec{i}}{\Delta t} = \frac{-2 \cdot 3 \cdot 10 \cdot \cos 30}{0{,}2}\,\vec{i}$$

$$= -260\,\vec{i}\ N$$

2- An estimated force–time curve for a baseball struck by a bat is shown in Figure. From this curve, determine
(a) the impulse delivered to the ball,
(b) the average force exerted on the ball, and
(c) the peak force exerted on the ball.





a) **Impuls is equal to the area under the curve.**

$$I = \frac{1}{2}\left(1{,}5 \cdot 10^{-3}\,s \times 18000\,N\right)$$
$$= 13{,}5\ N \cdot s$$

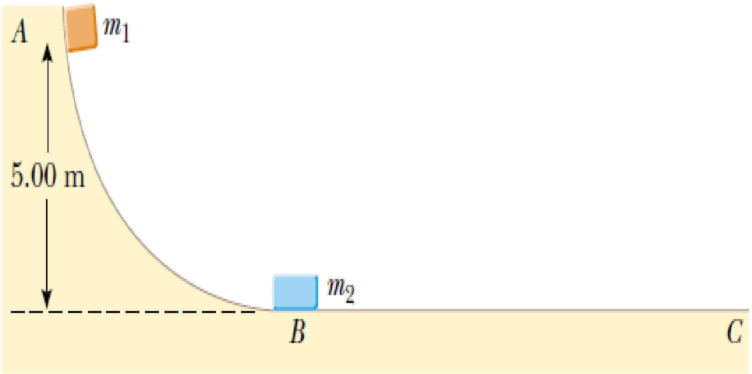
b) **Average Force:**

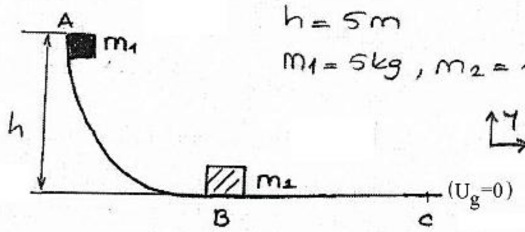
$$\vec{\bar{F}} = \frac{1}{\Delta t}\underbrace{\int_{t_i}^{t_s} \vec{F} \cdot dt}_{\vec{I}} = \frac{\vec{I}}{\Delta t}$$

$$\bar{F} = \frac{13{,}5\ N \cdot s}{1{,}5 \cdot 10^{-3}\,s} = 9 \cdot 10^{3}\ N$$

c) **From the graph** $F_{max} = 18000\ N$

3- Two blocks are free to slide along the frictionless wooden track *ABC* shown in Figure . The block of mass m_1 = 5.00 kg is released from *A*. Protruding from its front end is the north pole of a strong magnet, repelling the north pole of an identical magnet embedded in the back end of the block of mass m_2 = 10.0 kg, initially at rest. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.





$h = 5m$

$m_1 = 5kg$, $m_2 = 10$ kg

If we write conservation of energy for m_1: $\mu = 0 \Rightarrow \Delta E = 0$

$E_A = E_B$

$K_A^{\nearrow 0} + U_A = K_B + U_B^{\nearrow 0}$

$m_1 \cdot g \cdot h = \frac{1}{2} m_1 v_{B,1}^2 \rightarrow v_{B,1} = \sqrt{2gh} = \sqrt{2 \cdot 10 \cdot 5} = 10$ m/s

m_1 and m_2 collide elastically; In an elastic collision:

1) **Momentum is conserved:**

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad (v_2 = 0)$$

$$m_1 (v_1 - v_1') = m_2 v_2'$$

$$5 (v_1 - v_1') = 10 \cdot v_2' \Rightarrow \boxed{v_1 - v_1' = 2 v_2'}$$

2) **Kinetic energy is conserved:**

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \quad (v_2 = 0)$$

$$m_1 (v_1^2 - v_1'^2) = m_2 v_2'^2$$

$$5 (v_1^2 - v_1'^2) = 10 v_2'^2 \rightarrow \boxed{(v_1^2 - v_1'^2) = 2 v_2'^2}$$

$$(v_1 - v_1')(v_1 + v_1') = 2 v_2'^2$$

$$2 v_2' \cdot (v_1 + v_1') = 2 v_2'^2 \rightarrow \boxed{v_1 + v_1' = v_2'} \; (3)$$

If we combine Eq.(1) and Eq.(3)

$$v_1 - v_1' = 2 \cdot (v_1 + v_1') = 2 v_1 + 2 v_1'$$

$$v_1 = v_{B,1} = 10 \; m/s \rightarrow \quad -v_1 = 3 v_1' \rightarrow v_1' = -\frac{v_1}{3} = -\frac{10}{3} \; m/s$$

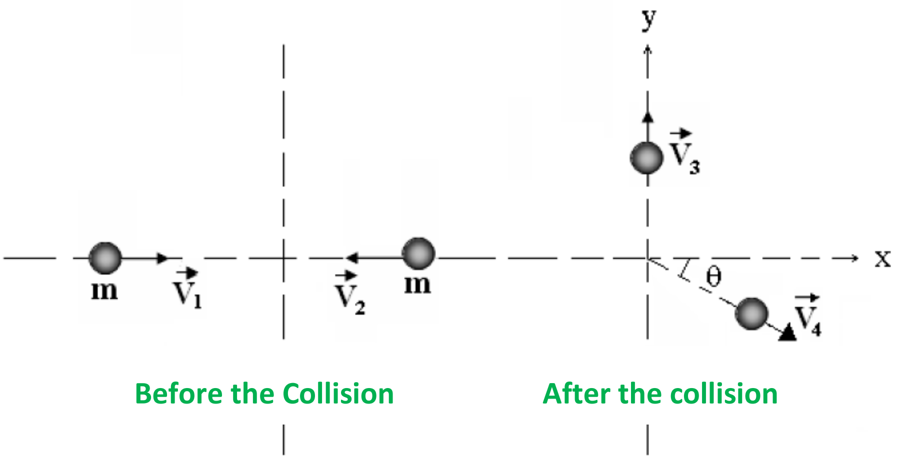
**** According to the results, m_1 moves backwards after the collision. Suppose that m_1 moves up to point A', If we write conservation of energy for this:**

$$E_B = E_{A'}$$

$$K_B + U_B = K_{A'} + U_{A'} \quad (U_B = 0, \; K_{A'} = 0)$$

$$\frac{1}{2} m_1 \cdot v_1'^2 = m_1 \cdot g \cdot h_{max} \rightarrow \quad h_{max} = \frac{v_1'^2}{2g} = \frac{(-10/3)^2}{2 \cdot 10} = \frac{5}{9} \; m$$

4- The two balls with speeds v_1= 8 m/s, v_2= 5 m/s collides elastically as shown in Figure. After the collision, one of them moves through +y direction. Find the velocities of the balls after the collision in terms of unit vectors.



İnitial velocities of the balls:

$$\vec{v}_1 = (8\,m/s)\,\vec{i} \quad ve \quad \vec{v}_2 = (-5\,m/s)\,\vec{i}$$

From the conservation of Momentum:

$$m(8\,m/s)\vec{i} + m(-5\,m/s)\vec{i} = m\,v_3\,\vec{j} + m\,(v_{4x}\,\vec{i} + v_{4y}\,\vec{j})$$

x ve y bileşenlerini eşitlersek

$$v_{4x} = (8\,m/s) + (-5\,m/s) = 3\,m/s$$

$$0 = v_3 + v_{4y} \longrightarrow v_{4y} = -v_3$$

Since the collision is elastic, The kinetic energy is conserved:

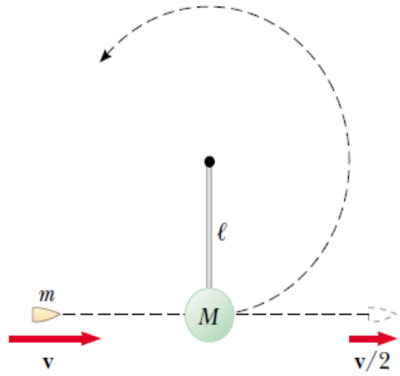
$$\frac{1}{2}m(8\,m/s)^2 + \frac{1}{2}m(-5\,m/s)^2 = \frac{1}{2}m\,v_3^2 + \frac{1}{2}m(v_{4x}^2 + v_{4y}^2)$$

$$64 + 25 = v_3^2 + 9 + v_3^2 \longrightarrow v_3 = \sqrt{40} \cong 6,3\,m/s$$

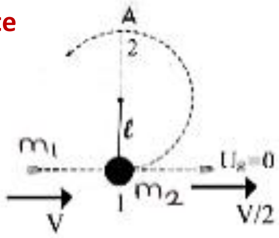
$$v_{4y} = -v_3 = -6,3\,m/s$$

$$\vec{v}_3 = 6,3\,\vec{j}\,, \quad \vec{v}_4 = v_{4x}\,\vec{i} + v_{4y}\,\vec{j} = (3\,m/s)\vec{i} - (6,3\,m/s)\vec{j}$$

5- As shown in Figure, a bullet of mass *m* and speed *v* passes completely through a pendulum bob of mass *M*. The bullet emerges with a speed of *v*/2. The pendulum bob is suspended by a stiff rod of length *l* and negligible mass. What is the minimum value of *v* such that the pendulum bob will barely swing through a complete vertical circle?



Bullet makes ,inelastic collision with the pendulum. If we write the conservation of momentum in the horizantal direction:



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v + 0 = m_1 \frac{v}{2} + m_2 v_2'$$

$$v = \frac{2 m_2 v_2'}{m_1} \quad \text{veya}$$

$$\boxed{v_2' = \frac{m_1 v}{2 m_2}} \quad (1)$$

The speed of the pendulum just after the collision.

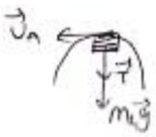
If we write the energy conservation for the pendulum:

$$E_1 = E_2$$

$$K_1 + U_1 = K_2 + U_2 \quad (U_1 = 0)$$

$$\frac{1}{2} m_2 v_2'^2 = \frac{1}{2} m_2 v_A^2 + m_2 g . 2l$$

$$v_2'^2 - v_A^2 = 4gl \quad (2)$$



$$\Sigma F_r = \frac{mv^2}{l}$$

$$T + m_2 g = \frac{m_2 v_A^2}{l} \quad (T = 0)$$

In order to obtain minimum speed at the top of the circle, T=0:

$$m_2 g = \frac{m_2 v_A^2}{l} \rightarrow \boxed{v_A^2 = gl} \quad (3)$$

If we write (1) and (3) into (2):

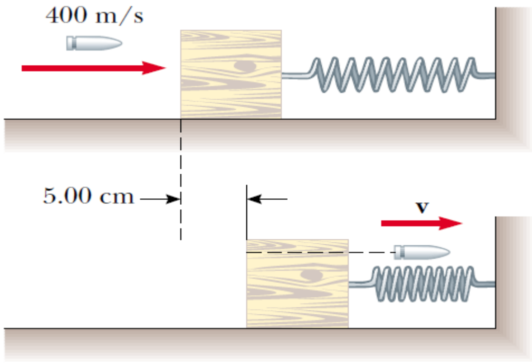
$$\left(\frac{m_1 v}{2 m_2}\right)^2 - gl = 4gl \Rightarrow \frac{m_1^2 v^2}{4 m_2^2} = 5gl$$

$$v = \frac{2 m_2}{m_1} \sqrt{5gl}$$

6- A 5.00-g bullet moving with an initial speed of 400 m/s is fired into and passes through a 1.00-kg block, as in Figure. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant 900 N/m. If the block moves 5.00 cm to the right after impact, find

(a) the speed at which the bullet emerges from the block and

(b) the mechanical energy converted into internal energy in the collision.



$m_1 = 5\cdot10^{-3}$ kg , $V_1 = 400$ m/s , $m_2 = 1$ kg , $k = 900$ N/m , $x = 5\cdot10^{-2}$ m

a) **Since the surface is frictionless, we can write conservation of energy:**

$$E_1 = E_2$$
$$K_1 + U_1 = K_2 + U_2$$
$$\frac{1}{2} m V_{2\,Blok}^2 + 0 = 0 + \frac{1}{2} k x^2$$
$$V_{Blok} = \sqrt{\frac{k x^2}{m_2}} = \sqrt{\frac{900\,(5\cdot10^{-2})^2}{1}}$$
$$V_{Blok} = 1{,}5 \text{ m/s}$$

In the Bullet-Block collision, The momentum is conserved:

$$m_1 \cdot V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \qquad V_2' = V_{blok} = 1{,}5 \text{ m/s}$$
$$(5\cdot10^{-3})(400) + 0 = 5\cdot10^{-3}\cdot V_1' + 1\cdot(1{,}5)$$
$$V_1' = 100 \text{ m/s}$$

b) **In the Bullet-Block collision, Energy is not conserved:**

$$\Delta E = E_2 - E_1$$
$$= (K_2 + U_2) - (K_1 + U_1)$$

veya $\Delta E = \Delta K + \Delta U$

$$= \frac{1}{2} m (V_1'^2 - V_1^2) + \frac{1}{2} k x^2$$

$K_1 = \frac{1}{2} m_1 V_1^2$

$U_1 = 0$

$K_2 = \frac{1}{2} m V_1'^2$

$U_2 = \frac{1}{2} k x^2$

$$\Delta E = \frac{1}{2} 5\cdot10^{-3}(100^2 - 400^2) + \frac{1}{2}\cdot 900\,(0.05)^2$$

$$\Delta E = -374 \text{ J}$$