1. Consider an electric field in the constant direction is perpendicular to plane of a circle with radius $R$. The magnitude of electric field at a distance $r$ from the center of the circle is 

$$E_0 \left[ 1 - \frac{r}{R} \right].$$ 

Determine the electric flux through the circle.

\[
\Phi = \int \mathbf{E} \cdot d\mathbf{A} = E \int dA = E_0 \left( 1 - \frac{r}{R} \right) 2\pi r dr
\]

\[
\Phi = E_0 2\pi \int_{0}^{R} \left( 1 - \frac{r}{R} \right) r dr
\]

\[
\Phi = E_0 2\pi \left( \frac{r^2}{2} - \frac{r^3}{3R} \right)_{0}^{R}
\]

\[
\Phi = \pi E_0 \frac{R^2}{3}
\]
2. Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4 \, \text{(N/C)}$ as shown in Figure 1. Calculate the electric flux through

a) the vertical rectangular surface,

b) the slanted surface,

c) the entire surface of the box.

![Figure 1](image)

a) $\Phi_1 = EA_1 \cos \theta = 7.8 \times 10^4 \times (0.1 \times 0.3) \cos 180^\circ = -2.34 \, \text{Nm}^2/\text{C}$

b) $\Phi_2 = EA_2 \cos 60^\circ = 7.8 \times 10^4 \times (0.2 \times 0.3) \cos 60^\circ$

$\Phi_2 = 2.34 \, \text{Nm}^2/\text{C}$

The flux through the base 5, the front 3 and the back 4 surface of the box is zero. Because, the electric field vector is perpendicular to the surface.

$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5$

$\Phi_{net} = -2.34 + 2.34 = 0 \, \text{Nm}^2/\text{C}$
3. A closed surface with dimensions \( a=0.2 \, \text{m}, \, b=0.3 \, \text{m} \) and \( c=0.3 \, \text{m} \) is located as in Figure 2. The left edge of the closed surface is located at position \( x=a \). The electric field throughout the region is nonuniform and given by \( E = (1 + x^2) \, \text{N/C} \), where \( x \) is in meters.

a) Calculate the net electric flux leaving the closed surface.

b) What net charge is enclosed by the surface?

![Figure 2](image-url)
\[ \Phi_E = -(1 + a^2) \int dA_1 + \left[ 1 + (a+c)^2 \right] \int dA_2 \]

\[ \Phi_E = -(1 + a^2) \cdot ab + \left[ 1 + (a+c)^2 \right] \cdot ab \]

\[ \Phi_E = -ab - a^3 b + ab + a^3 b + 2a^3 bc + abc = abc (2a+c) \]

\[ \begin{align*} 
    a & = 0.2 \text{ m} \\
    b & = 0.3 \text{ m} \\
    c & = 0.3 \text{ m} 
\end{align*} \]

\[ \Phi_E = 12.6 \cdot 10^{-3} \text{ N m}^2 / \text{C} \]

b) \[ \Phi_E = \frac{q_{\text{net}}}{\varepsilon_0} \Rightarrow q = \varepsilon_0 \frac{\Phi_E}{\varepsilon_0} \]

\[ \varepsilon_0 = 8.85 \cdot 10^{-12} \text{ C}^2 / \text{Nm}^2 \]

\[ q_{\text{net}} = 8.85 \cdot 10^{-12} \cdot 12.6 \cdot 10^{-3} \]

\[ q_{\text{net}} = 1.12 \cdot 10^{-13} \text{ C} \]
4. Three infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure 3. The sheets have a uniform surface charge density \( \sigma_1 = +5(\mu C/m^2) \), \( \sigma_2 = -10(\mu C/m^2) \) and \( \sigma_3 = +15(\mu C/m^2) \), respectively.

Calculate the electric field at
a) I zone,
b) II zone,
c) III zone,
d) IV zone.

Figure 3
I zone: \[ \hat{\mathbf{E}}_I = \mathbf{E}_1(-i) + \mathbf{E}_2(i) + \mathbf{E}_3(-i) \]
\[ \hat{\mathbf{E}}_I = (-2.82 + 5.65 - 8.47) \times 10^5 i \]
\[ \hat{\mathbf{E}}_I = 5.64 \times 10^5 (-i) \text{ N/C} \]

II zone: \[ \hat{\mathbf{E}}_II = \mathbf{E}_1(i) + \mathbf{E}_2(i) + \mathbf{E}_3(-i) \]
\[ \hat{\mathbf{E}}_II = (2.82 + 5.65 - 8.47) \times 10^5 i \]
\[ \hat{\mathbf{E}}_II = 0 \]

III zone: \[ \hat{\mathbf{E}}_III = \mathbf{E}_1(i) + \mathbf{E}_2(-i) + \mathbf{E}_3(-i) \]
\[ \hat{\mathbf{E}}_III = (2.82 - 5.65 - 8.47) \times 10^5 i \]
\[ \hat{\mathbf{E}}_III = 11.30 \times 10^5 (-i) \text{ N/C} \]

IV zone: \[ \hat{\mathbf{E}}_IV = \mathbf{E}_1(i) + \mathbf{E}_2(-i) + \mathbf{E}_3(i) \]
\[ \hat{\mathbf{E}}_IV = (2.82 - 5.65 + 8.47) \times 10^5 i \]
\[ \hat{\mathbf{E}}_IV = 5.64 \times 10^5 (i) \text{ N/C} \]
5. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire as in Figure 4. The wire has a charge per unit length of $+\lambda$, and the cylinder has a net charge per unit length of $+2\lambda$. From this information, use Gauss's law to find the electric field in the regions

\begin{enumerate}
\item[(a)] $r<a$,
\item[(b)] $a<r<b$,
\item[(c)] $r>b$.
\end{enumerate}

\item[(d)] Determine the charge distribution of the cylindrical sheet.
c) \[ E(2\pi r L) = \frac{\lambda L + 2\lambda L}{\varepsilon_0} \]

\[ E = \frac{1}{2\pi \varepsilon_0} \frac{3\lambda}{r} \]

\[ E = \frac{6\kappa \lambda}{r} \quad \text{if } r > \lambda \]

d) \[ q_i = -2\lambda \]

Because, wire induces the inner surface of the cylinder

\[ q_{\text{cylinder}} = q_i + q_{\text{out}} \]

\[ \lambda \cdot L = -2\lambda + q_{\text{out}} \]

\[ 2\lambda L + \lambda L = q_{\text{out}} \]

\[ q_{\text{out}} = 3\lambda L \]
6. There is a +2Q point charge at the centre of an empty insulating sphere which carries +Q total charge and has charge density, ρ.

a) Find the electric fields for R<r<2R and r>2R regions in terms of k, Q, r, and R.

b) If the sphere is conductor, calculate the electric fields for R<r<2R and r>2R regions.
b) \[ R < r < 2R \] zone (1)
inside conductor
\[ E = 0 ; \quad q_{\text{in}} = (q_{\text{in}})_{\text{surface}} + 2Q \]
\[ q = -2Q + 2Q = 0 \]

\[ E(4\pi r^2) = \frac{q_{\text{in}}}{\varepsilon_0} = 0 \]
\[ E = 0 \]

for \[ r > 2R \] zone (2)
\[ E(4\pi r^2) = \frac{2Q + Q}{\varepsilon_0} \]

\[ E = 2k \frac{Q}{r^2} \]
7. A solid, insulating sphere of radius \( R \) has a nonuniform charge density \( \rho = \alpha r \) and a total charge \( +2Q \) (\( \alpha \) is a positive constant and \( r \) radial distance from origin). Concentric with this sphere is a charged \((+4Q)\), conducting shell sphere whose inner and outer radii are \( 2R \) and \( 3R \), as shown in Figure 5.

a) Find \( \alpha \) constant in terms of \( Q \) and \( R \).

Find the magnitude of the electric field in the regions in terms of \( k \), \( Q \), \( r \) and \( R \).

b) \( r < R \)

c) \( R < r < 2R \)

d) \( 2R < r < 3R \)

e) \( r > 3R \)
c) \[ E(4\pi r^2) = \frac{2Q}{E_0} \]
\[ E = \frac{1}{4\pi \varepsilon_0} \frac{2Q}{r^2} \quad E = 2k \frac{Q}{r^2} \quad R < r < 2R \]

d) \[ E(4\pi r^2) = \frac{2Q - 2Q}{E_0} \]
\[ E = 0 \quad 2R < r < 3R \]

e) \[ E(4\pi r^2) = \frac{4Q + 2Q}{E_0} \]
\[ E = 6k \frac{Q}{r^2} \quad r > 3R \]
8. A point charge $q$ locates at the centre of a cylinder with radius $a$ and height $2h$ (see Figure 7). Show that the electric flux through the lateral surface of the cylinder is given by $\frac{\sqrt{2}}{2} \frac{q}{\varepsilon_0}$.

\[ \Phi_E = \frac{\sqrt{2}}{2} \frac{q}{\varepsilon_0} \]

**Figure 7**
\[ \Phi_E = 2\pi n^2 k q \int \cos^3 \theta \, \frac{\alpha \sec \theta \, d\theta}{\alpha^2} \]

\[ \Phi_E = 2\pi n k q \int_{-\pi/4}^{\pi/4} \cos \theta \, d\theta \]

\[ \Phi_E = 2\pi n k q \sin \theta \bigg|_{-\pi/4}^{\pi/4} \]

\[ \Phi_E = 2\pi n k q \left[ \sin \frac{\pi}{4} - \sin \left(-\frac{\pi}{4}\right) \right] \]

\[ \Phi_E = 2\pi n k q \left[ \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right) \right] \]

\[ \Phi_E = 2\pi n k q \sqrt{2} \]

\[ \Phi_E = 2n \frac{1}{4\pi \varepsilon_0} q \sqrt{2} \]

\[ \Phi_E = \frac{\sqrt{2}}{2} \frac{q}{\varepsilon_0} \]

\[ \sec \theta = \frac{1}{\cos \theta} \]

\[ \begin{align*}
y &= -a; \quad y = atg \theta \\
-a &= atg \theta \\
tg \theta &= -1 \\
\theta &= -\pi/4 \\
\hline
y &= a; \quad y = atg \theta \\
-a &= atg \theta \\
tg \theta &= 1 \\
\theta &= \pi/4
\end{align*} \]

\[
\text{integral zone}
\]