1. An air-filled capacitor consists of two parallel plates, each with an area of 200 cm$^2$, separated by a distance of 0.4 cm.
   a) Calculate the capacitance.
   b) If the capacitor had been connected to a 500-V battery, calculate the charge on each plate, the stored energy, the electric field between the plates and the energy density of the capacitor.
   c) If air had been replaced with a liquid of dielectric constant $\kappa = 2.6$, how much charge would have been flowed to the capacitor from the 500-V battery?
2. For the system of capacitors shown in Figure 1, 
   a) Find the total energy stored by the group.  
   b) When the discharge takes place on $C_3$ capacitor to convert to a conductor, how much charge and potential on $C_1$ would have been changed?

![Figure 1](image)

\[ C_1 = 10 \mu F \quad C_2 = 6 \mu F \quad C_3 = 4 \mu F \]
\[ \Delta V = 100 V \]

Due to the charges on capacitors connected in series are the same:

\[ Q = \frac{1}{2} C_e (\Delta V) \]
\[ Q = 3.2 \times 10^6 \times 100 \]
\[ Q = 3.2 \times 10^4 (c) \]

\[ U = \frac{1}{2} 3.2 \times 10^{-6} \times 100^2 \]
\[ U = 1.6 \times 10^{-3} (c) \]

After turning to the conductor,  
Potential difference on $C_3$ is equal to zero  
Initial potential difference on $C_1$ is equal to $\Delta V_{1i} = 20 (V)$  
After $C_3$ capacitor turns to the conductor, final potential difference on $C_1$ is equal to $\Delta V_f = 20 (V)$

\[ \Delta q = q_{1i} - q_{1f} \]
\[ \Delta q = (10 - 2) \times 10^{-6} ; \quad \Delta q = 8 \times 10^{-4} (c) \]
3. A parallel-plate capacitor has a plate separation of 1.2 cm and a plate area of 0.12 m². The plates are charged to a potential difference of 120 V and disconnected from the source. A dielectric slab having thickness 0.4 cm and a dielectric constant of $\kappa = 2$ is inserted exactly halfway between the plates as shown in Figure 2.

a) What is the capacitance before the dielectric being placed?

b) Calculate the capacitance after the slab is introduced using these equations:

$$C = \frac{Q}{\Delta V} \quad \text{and} \quad \Delta V = V_b - V_a = -\int_a^b E \cdot d\hat{s}$$

c) Find the charge on the plates. Determine the electric fields in the region of the dielectric and absence of the dielectric?
4. A conducting spherical shell has inner radius $a$ and outer radius $c$. The space between these two surfaces is filled with a dielectric for which the dielectric constant is $\kappa_1$ between $a$ and $b$, and $\kappa_2$ between $b$ and $c$ (Figure 3). Determine the capacitance of this system.

**Electric field in the region between the conductors,**

(i) $c < r < a$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$\mathbf{E} \cdot (4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\mathbf{E} = k \frac{Q}{r^2}$$

For the region with dielectric $\kappa_1$

$$V_b - V_a = \Delta V_{ab} = - \int_a^b \mathbf{E} \cdot d\mathbf{s}$$

$$\Delta V_{ab} = - \int_a^b k \frac{Q}{r^2} dr = - kQ \left[ -\frac{1}{r} \right]_a^b = kQ \left( \frac{1}{b} - \frac{1}{a} \right)$$

$$\Delta V_{ab} = kQ \left( \frac{a - b}{a b} \right) \quad a - b < 0 \quad \Delta V_{ab} < 0$$

For the region with dielectric $\kappa_2$

$$V_c - V_b = \Delta V_{bc} = - \int_b^c \mathbf{E} \cdot d\mathbf{s}$$

$$\Delta V_{bc} = - \int_b^c k \frac{Q}{r^2} dr = - kQ \left[ -\frac{1}{r} \right]_b^c = kQ \left( \frac{1}{c} - \frac{1}{b} \right)$$

$$\Delta V_{bc} = kQ \left( \frac{b - c}{b c} \right) \quad b - c < 0 \quad \Delta V_{bc} < 0$$

$$C = \frac{Q}{|\Delta V|} ; \quad C_1 = \kappa_1 \frac{Q}{|\Delta V_{ab}|} = \kappa_1 \frac{a b}{k(b - a)}$$

$$C_2 = \kappa_2 \frac{Q}{|\Delta V_{bc}|} = \kappa_2 \frac{b c}{k(c - b)}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} ; \quad \frac{1}{C} = \frac{1}{\kappa_1 \kappa_2 \frac{a b c}{4\pi \varepsilon_0}}$$

$$C = \frac{\kappa_1 \kappa_2 ab c (4\pi \varepsilon_0)}{\kappa_1 ab - a c} + ac (\kappa_1 - \kappa_2)$$

**Figure 3**
5. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure 4. You may assume that $\ell \gg d$

a) Find an expression for the capacitance of the device in terms of the plate area $A$ and $d, \kappa_1, \kappa_2$ and $\kappa_3$.

b) Calculate the capacitance using the values $A = 3\text{cm}^2, d = 1.5\text{mm}, \kappa_1 = 6, \kappa_2 = 3, \kappa_3 = 5$ and $\Delta V = 16V$.

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Figure 4

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\[ C_1 = \kappa_1 \varepsilon_0 \frac{A/2}{d} \quad ; \quad C_2 = \kappa_2 \varepsilon_0 \frac{A/2}{d/2} \quad ; \quad C_3 = \kappa_3 \varepsilon_0 \frac{A/2}{d/2} \]

\[
\left( \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right)
\]

\[
C_e = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3} \right)
\]

Using the given values:

\[
C_e = \frac{8.85 \cdot 10^{-12}}{4.5 \cdot 10^{-3}} \left( \frac{6}{2} + \frac{3.5}{3+5} \right) = 8.63 \cdot 10^{-12} \text{F}
\]

\[ C_e = 8.63 \mu \text{F} \]

\[
U = \frac{1}{2} C_e \left( \Delta V \right)^2 = \frac{1}{2} \cdot 8.63 \cdot 10^{-12} \cdot (16)^2 = 1.1 \cdot 10^{-9} \text{J}
\]

\[ U = 1.1 \mu \text{J} \]
6. A copper wire 2m long and 4mm in diameter carries a current of 6A. If the conductor is copper with a free charge density of \(8.5 \times 10^{28} \text{ (1/m}^3\text{)}\) and a resistivity of \(\rho = 1.6 \times 10^{-6} \Omega \text{cm}\), calculate,

a) the current density,

b) the electric field,

c) the resistance,

d) the average drift velocity of free electrons,

e) the power dissipated as heat in this wire. \((e = 1.6 \times 10^{-19} \text{ C}, \pi = 3)\)

\[\begin{align*}
2r &= 4 \text{ mm} = 4 \times 10^{-3} \text{ m} \\
l &= 2 \text{ m} \\
I &= 6 \text{ A} \\
n &= 8.5 \times 10^{28} \text{ (1/m}^3\text{)} \\
g &= 1.6 \times 10^{-5} \text{ m.cm} = 1.6 \times 10^{-6} \text{ m} \\
A &= \pi r^2 \\
A &= \pi (2 \times 10^{-3})^2 \\
A &= 1.26 \times 10^{-5} \text{ m} \\
\]
1. Material with uniform resistivity $\rho$ is formed into a wedge as shown in Figure 5. Find the resistance between face A and face B of this wedge.

Figure 5

\[ R = \frac{\rho}{\omega} \int_0^L \frac{dx}{y_1 + \frac{y_2 - y_1}{L} x} = \frac{\rho L}{\omega (y_2 - y_1)} \ln \left[ y_1 + \frac{y_2 - y_1}{L} x \right]_0^L \]

\[ R = \frac{\rho L}{\omega (y_2 - y_1)} \ln \left( \frac{y_1 + y_2 - y_1}{y_1} \right) \]

\[ R = \frac{\rho L}{\omega (y_2 - y_1)} \ln \left( \frac{y_2}{y_1} \right) \]
2. For the circuit in Figure 6, find
   a) the dissipated power for each resistance ($R_1$, $R_2$, and $R_3$).
   b) the power supplied by $\varepsilon_1$ and $\varepsilon_2$ generators.

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![Kirchhoff's Rules](image)

1. Junction rule \[ \sum_{\text{junction}} I = 0 \]
2. Loop rule \[ \sum_{\text{closed loop}} \Delta V = 0 \]

For abe\(\text{f}\)a loop:
\[-I_1R_3 - I_1R_4 + \varepsilon_4 = 0 \]
\[-4I_3 - 5I_4 + 3 = 0 \quad (2)\]

For bc\(\text{d}\)eb loop:
\[-I_2R_2 - \varepsilon_2 + I_4R_4 = 0 \]
\[-2I_2 - 1 + 5I_4 = 0 \quad (3)\]

For b junction:
\[ I_3 = I_1 + I_2 \quad (4)\]

From (4), (2) and (3) equations; \[ I_1 = \frac{5}{19} \text{ (A)} \], \[ I_2 = \frac{3}{19} \text{ (A)} \], \[ I_3 = \frac{8}{19} \text{ (A)} \]

\[ P_{R_1} = I_1^2R_1 = \frac{125}{361} \text{ (w)} \]
\[ P_{R_2} = I_2^2R_2 = \frac{18}{361} \text{ (w)} \]
\[ P_{R_3} = I_3^2R_3 = \frac{256}{361} \text{ (w)} \]

\[ P_{\varepsilon_1} = \varepsilon_1I_3 = \frac{24}{19} \text{ (w)} \]
\[ P_{\varepsilon_2} = \varepsilon_2I_2 = \frac{3}{49} \text{ (w)} \]
3. In the circuit the capacitor is uncharged, the switch \( S \) closes at \( t = 0 \), as in Figure 7.
   a) Express the current \( I \) in the circuit as functions of time and sketch \( I = f(t) \) graph.
   b) After the circuit becomes the steady-state, the switch \( S \) is opened. Find the time interval required for the charge on the capacitor to fall to one-second its initial value.

\[
\begin{align*}
\text{for b junction:} & \quad I = I_1 + I_2 \\
\text{for acdf a loop:} & \quad E = I_2 \cdot 2R = 0 \quad I_2 = \frac{E}{2R}
\end{align*}
\]

\[
I(t) = I_1(t) + I_2
\]

\[
I(t) = \frac{E}{R} e^{-t/RC} + \frac{E}{2R}
\]

\[
t = 0 \quad \Rightarrow \quad I(0) = \frac{E}{R} + \frac{E}{2R} = \frac{3}{2} \frac{E}{2R}
\]

\[
t \to \infty \quad \Rightarrow \quad I(\infty) = \frac{E}{2R} \quad (e^{-\infty} = 0)
\]

\[
q(t) = Q e^{-t/RC}
\]

\[
q(t) = \frac{Q}{2} \quad \Rightarrow \quad \frac{Q}{2} = Q e^{-t/RC} \quad \ln\left(\frac{1}{2}\right) = -\frac{t}{3RC}
\]

\[
t = -3RC \ln\left(\frac{1}{2}\right)
\]

\[
t = 3RC \ln 2
\]
4. If no charges exist on the capacitor before switch $S$ is closed $t = 0$ as in Figure 8.
   a) Shortly after the switch $S$ is closed, find the currents $I_1$, $I_2$ and $I_3$.
   b) After the switch $S$ has been closed for a length of time sufficiently long, find the currents $I_1$, $I_2$ and $I_3$.
   c) After the switch $S$ has been closed for long time, find the potential difference between $a$ and $b$ points.
   d) Find the charge on the capacitor after the switch $S$ has been closed for long time.

![Figure 8](image-url)

\[
\text{for a junction:} \quad I_1 = I_L + I_3 \quad (4)
\]

\[
\text{for \; cdcb \; loop:} \quad -I_3 R_L + E_a - I_1 R_1 = 0
\]
\[
-8I_3 + 40 - 6I_4 = 0
\]
\[
6I_4 + 8I_3 = 40 \quad (2)
\]

\[
\text{for \; bafca \; loop:} \quad I_1 R_2 - E_a + E_b + I_4 R_4 + I_2 R_3 = 0
\]
\[
6I_4 - 4I_2 + 2I_3 = 28 \quad (3)
\]

from (4), (2) and (3) equations: $I_1 \equiv 3.7(A)$ $I_2 \equiv 4.5(A)$ $I_3 \equiv 2.2(A)$

b) In the steady-state, there is no current through the capacitor (ab).

\[
I_1 = 0
\]

\[
\text{for \; cebdc \; loop:} \quad -I_2 R_3 - I_2 R_4 - I_2 R_3 = 0
\]
\[
-3I_2 - I_2 - 2I_3 = 0
\]

\[
I_2 = -1(A) \quad I_3 = 1(A)
\]

c) \quad V_a - I_3 R_4 = V_b
\]
\[
V_a = V_b = 8(V)
\]

d) \quad V_a - E_a + V_c = V_b
\]
\[
V_a - V_b = E_c - V_c
\]
\[
8 = 40 - V_c
\]
\[
V_c = 32(V)
\]

\[
Q = CV_c
\]
\[
Q = 2.4 \times 10^6 \text{ Coulomb}
\]
\[
Q = 64 \times 10^6 \text{ Coulomb}
\]
5. In the circuit shown in Figure 9,
   a) After the switch $S$ has been closed for a length of time sufficiently long, find the currents on each resistance.
   b) Find the charges for each capacitors and the dissipated power on the resistance $R_2$.
   c) If the switch $S$ is opened, find the time constant of the discharging circuit.
   d) After the switch $S$ is opened, write the current on the resistance $R_2$ as a function of time.

![Diagram of the circuit](image-url)