1. A conductor wire carrying a constant current $I$ is in a uniform magnetic field $(\vec{B})$ oriented perpendicularly into the plane of the Figure 1. Find the components of the magnetic force on the wire.

**Figure 1**

\[
\vec{F}_a = I \hat{z} \times \vec{B}
\]

for 4. and 4. parts:

\[
\vec{F}_i = \vec{F}_u = I \hat{d} \times \vec{B}
\]

\[
\hat{d} = d(-\hat{j})
\]

\[
\vec{B} = B(-\hat{z})
\]

\[
\vec{F}_a = \vec{F}_u = I d(-\hat{j}) \times B(-\hat{z})
\]

\[
\vec{F}_i = I \hat{d} B \hat{z}
\]

for 3. part:

\[
\vec{F}_3 = I \hat{R} \times \vec{B}
\]

\[
\hat{R} = R \hat{i}
\]

\[
\vec{B} = B(-\hat{z})
\]

\[
\vec{F}_3 = I R \hat{i} \times B(-\hat{z})
\]

\[
\vec{F}_i = I R \hat{B} \hat{j}
\]

for 2. part:

\[
d\vec{F}_L = I d\hat{d} \times \vec{B}
\]

\[
ds = R d\theta
\]

\[
d\vec{s} = ds \cos \theta (-\hat{i}) + ds \sin \theta (-\hat{j})
\]

\[
d\vec{s} = R \cos \theta d\theta (-\hat{i}) + R \sin \theta d\theta (-\hat{j})
\]

\[
\vec{F}_i = I \left( \int_0^{\pi} R \cos \theta d\theta (-\hat{i}) + \int_0^{\pi} R \sin \theta d\theta (-\hat{j}) \right) \times \vec{B}(-\hat{z})
\]

\[
\vec{F}_i = IR (-\hat{i} - \hat{j}) \times \vec{B}(-\hat{z})
\]

\[
\vec{F}_i = I R \hat{B} (-\hat{i} - \hat{j})
\]

\[
\sum \vec{F} = \vec{F}_i + \vec{F}_u + \vec{F}_3 + \vec{F}_L
\]

\[
\sum \vec{F} = I B (2d + R) \hat{i}
\]
2. A closed rectangular loop carrying a constant current $I$ lies in the $xy$-plane ($b$ is parallel to $x$-axis and $a$ is parallel to $y$-axis) as shown in Figure 2. The magnetic field is not uniform and given by $\mathbf{B} = \alpha y \hat{k}$ ($\alpha$ is a constant). Find the components of the net magnetic force on the loop.

![Figure 2]

For 1. part:

$$\mathbf{F}_1 = I \int d\mathbf{r} \times \mathbf{B}$$

$$d\mathbf{r}_1 = dx (-\hat{i})$$

$$\mathbf{B} = \alpha y \hat{k} = \alpha \hat{k}$$

For 2. part:

$$\mathbf{F}_2 = I \int d\mathbf{r}_2 \times \mathbf{B}$$

$$d\mathbf{r}_2 = dy \hat{j}$$

$$\mathbf{B} = \alpha y \hat{i}$$

For 3. part:

$$\mathbf{F}_3 = I \int d\mathbf{r}_3 \times \mathbf{B}$$

$$d\mathbf{r}_3 = dx \hat{i}$$

$$\mathbf{B} = \alpha (c+a) \hat{k}$$

For 4. part:

$$\mathbf{F}_4 = I \alpha \left( \frac{a^2 + 2ac}{2} \right) \hat{i}$$

Part 4 is same magnitude with 2. part but in the reverse direction.

$$\mathbf{F}_u = I \alpha \left( \frac{a^2 + 2ac}{2} \right) (-\hat{i})$$

For 5. part:

$$\mathbf{F}_5 = I \int d\mathbf{r}_5 \times \mathbf{B}$$

$$d\mathbf{r}_5 = dx \hat{i}$$

$$\mathbf{B} = \alpha (c+a) \hat{k}$$

$$\mathbf{F}_5 = I \alpha \hat{k} (c+a) (-\hat{j})$$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{F}_5$$

$$\mathbf{F} = I \alpha a \hat{b} (-\hat{j})$$
3. A rectangular loop consists of $N=100$ closely wrapped turns and has dimensions $a = 1 \text{ m}$ and $b = 2$. The loop is hinged along the $z$ axis, and its plane makes an angle $60^\circ$ with the $x$ axis (Figure 3).

(Neglect the magnetic field exerted by the loop)

a) What are the magnitude and direction of the torque on $KL$ part of the loop exerted by a uniform magnetic field $B=100 \text{ mT}$ directed along the $y$ axis when the current is in $l=10 \text{ A}$ the direction shown?

b) Find the dipole moment of the loop and the torque on the loop exerted by the magnetic field.

c) Calculate the magnetic potential energy of the loop.

\[ \mathbf{F}_B = I \mathbf{L} \times \mathbf{B} \]
\[ \mathbf{F}_B = I \mathbf{L} \times \mathbf{B} \]
\[ \mathbf{F}_B = 10 \times \left[ (2\hat{i}) \times (0,1,0) \right] \]
\[ \mathbf{F}_B = -200 \hat{j} \text{ (N)} \]

\[ \mathbf{M} = N I A \]
\[ M = 100 \times 10 \times 2 = 2 \times 10^3 \text{ (A.m$^2$)} \]
\[ \mathbf{M} = \mu \cos 30 \hat{i} - \mu \sin 30 \hat{j} \]
\[ \mathbf{M} = 2 \times 10^3 \left( \frac{\sqrt{3}}{2} \hat{i} - \frac{1}{2} \hat{j} \right) \]
\[ \mathbf{M} = 10^3 \left( \sqrt{3} \hat{i} - 3 \hat{j} \right) \text{ (A.m$^2$)} \]
\[ \mathbf{Z} = \mathbf{M} \times \mathbf{B} \]
\[ Z = 10^3 \left( \hat{i} \times \hat{j} \right) \times 0.1 \hat{k} \]
\[ Z = 10^3 \times \sqrt{3} \hat{k} \text{ (N.m)} \]

\[ U = -\mathbf{M} \cdot \mathbf{B} \]
\[ U = 100 \left( \sqrt{3} \hat{i} - 3 \hat{j} \right) \cdot 0.1 \hat{k} \]
\[ U = 100 \text{ (J)} \]

or

\[ U = -MB \cos \theta \]
\[ U = -2 \times 10^3 \times 0.1 \cos 120^\circ \]
\[ U = 100 \text{ (J)} \]
4. A closed loop carrying a constant current is in a uniform magnetic field given by \( \vec{B} = \hat{i} - 2\hat{j} + \hat{k} \) (Figure 4). Ignoring the magnetic field exerted by the loop, find

a) the magnetic force vector on MN part of the loop.

b) the components of the magnetic dipole moment of the loop.

c) the torque acting on the loop and the magnetic potential energy of the loop.

![Figure 4]
5. The number of turns of a coil having a section of 6cm² is 50. When the coil is placed in a uniform magnetic field of 0.2T, the maximum torque is $3 \times 10^{-5} N.m$.
   a) Find the magnitude of current on the coil.
   b) How much work is done to rotate the coil with the angle of $180^\circ$ in the magnetic field?

\[ A = 6 \text{ cm}^2 = 6 \times 10^{-4} \text{ m}^2 \]
\[ N = 50 \]
\[ B = 0.2 \text{ T} \]
\[ \tau = 3 \times 10^{-5} \text{ N.m} \]
\[ \mu = 4.5 \times 10^{-4} \text{ (A.m)} \]
\[ I = 5.10^{-3} \text{ (A)} \]

\[ I = 5 \text{ (mA)} \]

a)
\[ \tau = \mu B (\theta = 30^\circ) \]
\[ \tau = 4.5 \times 10^{-4} \times 0.2 \times \frac{\sqrt{3}}{2} \]
\[ \tau = 3 \times 10^{-5} \times 0.12 \]
\[ \tau = 3 \times 10^{-5} \text{ N.m} \]

b)
\[ W = U_f - U_i \]
\[ W = \mu B \cos \theta \]
\[ W = 2 \mu B \cos \theta \]
\[ W = 6 \times 10^{-5} \cos \theta \]
6. A particle having charge $q$ and mass $m$ enters into a velocity selector as to be perpendicular to the electric and the magnetic fields (Figure 5). The particle moves with a constant speed in the velocity selector. The particle reaches point $P_2$ only effect on same magnetic field ($B_{in}$) from point $P_1$ by orbital motion. ($B_{in} = 0.2 \ T$; $E = 4 \times 10^5 \ V/m$; $r = 0.1 \ m$; $\pi = 3$)

a) Find the velocity of the particle.
b) Determine the direction of the electric field and the sign of the charged particle according to the given coordinate system.
c) Calculate $q/m$ ratio.
d) Find the arrival time of the particle from point $P_1$ to point $P_2$.

Figure 5
7. A uniform magnetic field of magnitude 0.1 T is directed along the positive x axis. A positron with an energy of 2 keV enters the field along a direction that makes 85° with the x axis (Figure 6). The motion of the particle is expected to be a helix. Calculate
a) the period of the positron
b) the pitch \( p \) of helix.
c) the radius \( r \) of the trajectory.

\( m = 9.1 \times 10^{-31} \text{ kg}, \quad q = 1.6 \times 10^{-19} \text{ C} \)

\[ \begin{align*}
\text{a) } & qvB = m \frac{\nu}{\tau} \\
\tau &= \frac{m}{qB} \\
T &= \frac{2\pi m}{qB} \\
T &= \frac{2\pi \cdot 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \cdot 0.10} \\
T &= 3.57 \times 10^{-10} \text{(s)}
\end{align*} \]

\[ \begin{align*}
\text{b) } & p = T \cdot \nu \\
p &= \frac{2\pi m}{qB} \cdot \nu \cos 85° \\
p &= 3.57 \times 10^{-10} \cdot 2.67 \times 10^{-3} \cos 85° \\
p &= 8.3 \times 10^{-17} \text{(m)}
\end{align*} \]

\[ \begin{align*}
\text{c) } & r = \frac{mv}{qB} \\
r &= \frac{mv}{1.6 \times 10^{-3} \cdot 9.1 \times 10^{-31} \cdot 2.67 \times 10^{-3} \sin 85°}{0.10} \\
r &= 1.5 \times 10^{-2} \text{(m)}
\end{align*} \]

Figure 6